CBCS SCHEME

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21MAT21

Second Semester B.E. Degree Examination, June/July 2023
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x+y+z)\,dx\,dy\,dz$. **(06 Marks)**

b. Evaluate $\int_{0}^{1}\int_{x}^{\sqrt{x}} xy\,dy\,dx$ by changing the order of integration. **(07 Marks)**

c. Prove that $\pi(\frac{1}{2})=\sqrt{\pi}$, using definition of Gama function. **(07 Marks)**

OR

2 a. Evaluate $\int_{0}^{\infty}\int_{0}^{\infty} e^{-(x^2+y^2)}\,dx\,dy$ by changing into polar coordinates. **(06 Marks)**

b. Find the area between the parabolas $y^2=4ax$ and $x^2=4ay$ by using double integration. **(07 Marks)**

c. Show that $\beta(m,n)=\dfrac{\prod(m)\prod(n)}{\prod(m+n)}$. **(07 Marks)**

Module-2

3 a. Find the directional derivative of $\phi=x^2yz+4xz^2$ at the point $(1,-2,-1)$ along $2\hat{i}-\hat{j}-2\hat{k}$. **(06 Marks)**

b. If $\vec{F}=\nabla(xy^3z^2)$, find $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$ at the point $(1,-1,1)$. **(07 Marks)**

c. If $\vec{F}=(x+y+az)\hat{i}+(bx+2y-z)\hat{j}+(x+cy+2z)\hat{k}$, find a, b, c such that $\operatorname{curl}\vec{F}=0$. **(07 Marks)**

OR

4 a. If $\vec{F}=xy\hat{i}+yz\hat{j}+zx\hat{k}$, evaluate $\int_C \vec{F}\cdot d\vec{v}$ where 'c' is the curve represented by $x=t$, $y=t^2$, $z=t^3$, $-1\le t\le 1$. **(06 Marks)**

b. Using Green's theorem, evaluate $\int_C (xy+y^2)dx+x^2dy$, where 'c' is bounded by $y=x$ and $y=x^2$. **(07 Marks)**

c. Apply Stoke's theorem to evaluate $\iint \operatorname{curl}\vec{F}\cdot\hat{n}\,ds$ where $\vec{F}=(x^2+y^2)\hat{i}-2xy\hat{j}$ taken around the rectangle bounded by the lines $x=\pm a$, $y=0$ and $y=b$. **(07 Marks)**

Module-3

5 a. Form a partial differential equation by eliminating arbitrary function from $Z = f(x + at) + g(x - at)$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$ when x = 0 and z = 0 when y is an odd multiple of $\frac{\pi}{2}$. **(07 Marks)**

c. Derive one dimensional heat equation. **(07 Marks)**

OR

6 a. Form a partial differential equation by eliminating arbitrary constant from $Z = (x - a)^2 + (y - b)^2$. **(06 Marks)**

b. Solve $(y - z)p + (z - x)q = x - y$. **(07 Marks)**

c. Solve $\frac{\partial^2 z}{\partial y^2} = z$ given that when $y = 0$, $z = e^x$ and $\frac{\partial z}{\partial y} = e^x$. **(07 Marks)**

Module-4

7 a. The area of a circle (A) corresponding to diameter (D) is given below:

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the area corresponding to diameter 105 using an appropriate interpolation formula. **(06 Marks)**

b. Find a real root of $x^3 - 2x - 5 = 0$ using Regula-Falsi method correct to 3 decimal places whose root lies between 2 and 2.5. **(07 Marks)**

c. Evaluate $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ by taking 7 ordinates by Simpson's 1/3rd rule. **(07 Marks)**

OR

8 a. Use Newton's divided difference formula to find f(4) given the data:

x	0	2	3	6
f(x)	–4	2	14	158

(06 Marks)

b. Use Newton-Raphson method to find a real root of $x\sin x + \cos x = 0$ near $x = \Pi$. Carry out the iterations upto 4 decimal places. **(07 Marks)**

c. Use Lagrange's interpolation formula to find y when x = 35 to the following data:

x	25	30	40	60
f(x)	50	55	70	95

(07 Marks)

Module-5

9 a. Use the Taylor series method to find y(0.2) from $\frac{dy}{dx} = y + \sin x$, $y(0) = 1$. **(06 Marks)**

b. Use Runge-Kutta method of order 4, find y at x = 0.1, given that $\frac{dy}{dx} = 3e^x + 2y$, $y(0) = 0$ with $h = 0.1$. **(07 Marks)**

c. Apply Milne's predictor-corrector method, to find y(1.4) from $\frac{dy}{dx} = x^2 + \frac{y}{2}$ given that $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$, $y(1.3) = 2.7514$. **(07 Marks)**

OR

10 a. Use modified Euler's method to solve $\frac{dy}{dx} = x^2 + y$ with $y(0) = 1$, $h = 0.05$ at $x = 0.1$. **(06 Marks)**

b. Use Taylor series method to find y(0.1) from $\frac{dy}{dx} = x^2 + y^2$ with $y(0) = 1$. **(07 Marks)**

c. Use Runge-Kutta method of 4th order, find y(0.1) given that $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0) = 1$ with $h = 0.1$. **(07 Marks)**

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